Probability of the Occurrence of Two Significant Earthquakes on the Same Date (of Different Years) Striking the Same Site: The Mexico City Case

by Miguel A. Jaimes and Adrián D. García-Soto

ABSTRACT

On the afternoon of 19 September 2017, just a couple of hours after an earthquake drill, an intense M_w 7.1, intermediate-depth, normal-fault earthquake struck Mexico City and neighboring states causing destruction and 369 deaths. This earthquake occurred on exactly the same date as the well-known 1985 M_w 8.1 subduction earthquake that also struck Mexico City causing severe damage, structure collapses and more than 10,000 deaths. There was a difference of ~6 hrs between the hours of these historical seismic events; the 1985 event occurred at 7:17 (CST), and the 2017 event at 13:14 (CDT).

Classic probability theory, probabilistic seismic hazard assessment (PSHA), and information on the correlation between deaths and seismic intensity are used to compute the probability of two significant earthquakes occurring on the same date and striking the same site. The probabilities of events associated with two significant earthquakes in these circumstances can be very low, lower than events of a different kind that are usually considered as highly improbable. This may not necessarily be expected for an earthquake-prone region such as Mexico City, where significant events occur relatively frequently (approximately a 9% probability in any given year); for any given day the probability decreases to about 0.026%, and for the referred event involving two earthquakes, the probability can be even smaller (on the order of 10^{-5} to 10^{-8}), unless periods of observation of over a century are employed. Although this is expected, such probability is rarely (if at all) rigorously computed and not typically presented in an engaging way for students and the general public.

INTRODUCTION

The occurrence of two significant earthquakes on a specific date affecting the same location may seem very improbable. However, a recent earthquake, on 19 September 2017, occurred on exactly the same day of the year as another well-known major event in 1985. Some people now believe that major earthquakes in Mexico are more likely to occur on 19 September, but as our analysis will show, there is an extremely small probability to support this hypothesis. However, the choice of 19 September as the date for a national earthquake drill and Memorial Day for the victims of the 1985 and 2017 earthquakes is an excellent one, because many Mexicans have strong memories of these two events.

The 2017 earthquake caused 369 deaths: 228, 74, 45, 15, 6, and 1 in Mexico City, Morelos, Puebla, State of Mexico, Guerrero, and Oaxaca, respectively (CENAPRED, 2017, see Data and Resources; the acronym stands for the National Disasters Management and Prevention Agency of Mexico, in Spanish). It also left 5714 affected survivors, according to official sources (El Universal, 2018). To compute the probability of two seismic events on the same day of the year causing damage and death at the same site, we use formal probability theory and seismic hazard assessment tools (Esteva, 1967; Cornell, 1968) and also a proposed expression to relate seismic intensity and number of deaths (e.g., Jaimes and García-Soto, 2018). This expression can be used within the framework of formal risk analysis to assess the mean annual rate of exceedance of the loss of human lives. It can also be developed to define a significant earthquake (by quantitatively determining the number of fatalities as a function of seismic intensity), as reported somewhere in the Jaimes and García-Soto (2018) reference; for this article, an exceedance rate to delimit significant earthquakes is selected as explained below.

Historical Perspective

It is interesting to note that what is now known as Mexico City was founded in 1325 (Tenochtitlán at the time) over a little island in the Texcoco Lake, according to the Mexicayotl chronicle (Alvarado, 1994). The city soon became the economic and cultural center of the region. In three more years (2021), Mexico City will commemorate the 500th anniversary of its refoundation since the Spanish conquest. The first earthquake ever mentioned in historical sources occurred in 1455, when it was cited by the Indigenous people of that region and also by the Spaniard conquers. Tenochtitlán was devastated and taken by the Spaniard invaders in 1521, who recorded that

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"In the Valley of Mexico, this year and the previous two ones, there were freezes and lack of corn. In 1455: ...there was also an earthquake and the soil cracked and the chinampas [small structures for lake agriculture] collapsed; and people rented themselves to others because of the famine" (García and Suárez, 1996, p. 71).

Several centuries later, in 1910, after more than 30 yrs in power, the Mexican Dictator Porfirio Díaz was celebrating the 100th anniversary of the War of Independence in the country and inaugurating majestic monuments and buildings, unaware that the Mexican Revolution was about to explode that same year and end his cruel and bloody dictatorship. Porfirio Díaz also inaugurated the first seismological observatory in Tacubaya, Mexico City, in 1910 (García and Suárez, 1996). More recently, the first accelerometer was installed in Mexico City at the CU station in 1962, recording the first earthquake in 1964. Since then, the CU station is the reference site to assess ground motion at hard-soil locations and to estimate the seismic response in soft soil on the lakebed of Mexico City (e.g., Singh *et al.*, 1988; Ordaz *et al.*, 1989; Reinoso and Ordaz, 1999). A large earthquake that occurred in relatively modern times and that is associated with fatalities was unfortunately not recorded at the CU station, because it occurred in 1957. The previous historical descriptions, and the referred dates, are to be used later for the probabilistic analysis.

19 September was already officially a memorial national day for "civil protection" (i.e., management and prevention of disasters). Since then, not only drills but also remembrance events have been carried out on the same date, because of the exemplary solidarity of the Mexican people, surpassing that of the government (it was the case again in 2017) and because it prompted a "grassroots movement to challenge corruption and secure low-cost housing" (The Guardian, 2015). In fact, two important U.S. researchers (Alfredo Ang from the University of California, Irvine, and Janise Rodgers, Chief Operating Officer of GeoHazards International) personally experienced an earthquake drill that had taken place at a meeting, the "Earthquake anniversary forum" in Mexico City (on the 32nd anniversary of the 1985 earthquake), when the new earthquake struck the city, just about 2 hrs after the drill (Rodgers, 2017).

An additional aspect on the importance of the occurrence of significant earthquakes on specific dates is that certain days may be more critical than others in the case of strong seismic activity (e.g., a special public holiday, when people are gathered in places that could be vulnerable to earthquakes). It could be argued that it is the holiday itself, and not the possibility of two events occurring on the same date, that really matters.

To elaborate on this point, we consider a catastrophic event of a different kind, the landslide that occurred at La Pintada, Guerrero, in Mexico in 2013 (Alcántara-Ayala *et al.*, 2017). The landslide happened on 16 September (a national public holiday in Mexico, Independence Day), which increased the number of casualties: 78 fatalities, 8 missing persons, and 8 injured (Alcántara-Ayala *et al.*, 2017). If it is hypothetically considered that a scenario of a second tragic event could occur again on 16 September, striking the same site (and here the number of years which passed between one event and the other may also play a role), would this have implications in disaster management and prevention, since the probability of occurrence decreases? If the event occurred at a different date, leading to less fatalities, would this influence the decision making (e.g., relocation of the village, protection projects)? Moreover, if it happened again on the same day of the year, would it not have an impact at a social and psychological level (for instance, implying that people may consider it as a warning of a divinity to abandon the place, or simply people deciding not to celebrate on such a conspicuous date anymore)? It is also interesting to note that La Pintada is an archeological site with petroglyphs, suggesting past landslide activity (the petroglyphs themselves are on massive rocks that could have been mobilized by a landslide) (Alcántara-Ayala *et al.*, 2017). It is not possible to know whether similar events occurred exactly on 16 September (indeed long before the War of Independence in Mexico). However, the time window can be considered in the probability computed for catastrophic events, and in fact it does have an impact on the results, as we will show later. Other psychological effects (e.g., negative effects, such as the accentuation of more traumatic situations or positive effects, such as promoting awareness of the prevention) could also be of interest.

The issues implied above (psychological, social, cultural, etc.) are not properly answered, but they could be intuitively considered relevant. Furthermore, the previous discussion may persuade the reader that quantification of the probability of two events on coincidental dates could be important, even for unforeseen reasons.

The oddness of the described coincidental occurrence of major earthquakes may help to promote the need for earthquake drills in a more effective way, from the psychological and mass-media standpoints.

Estimating the Probability of an Odd Coincidence

The obtaining of a probability such as the one referenced above is rarely, if ever, undertaken, at least in a quantitative and rigorous way. The importance of using scientific tools to rigorously address simple questions about earthquakes has been pointed out in other studies (e.g., Hough, 2018). Here, we compute the probability of the occurrence of two earthquakes in different years but on the same date and striking the same site. The considered earthquakes are significant ones (large ones), in the sense that they are associated with death and destruction.

Young engineers and students from other fields will realize how basic probabilistic concepts and seismic information are used to answer simple questions as the one stated above (or others of a similar kind) by reading this article. They may try to reproduce the obtained probabilities or compute others for different scenarios concerning odd events for earthquakes. At the same time, the article is written in such a way that the general public could understand how improbable an event of this kind is but that coincidences do occur.

For clarity, we present the probability of the occurrence of two significant earthquakes in the three following steps:

1. use of classic probability theory (e.g., Benjamin and Cornell, 1970; Ang and Tang, 1975, 1984),
2. use of seismic hazard information and a measure to relate seismic intensity and deaths (e.g., Jaimes and García-Soto, 2018), and
3. a combination of the first two methods.

SOME APPROACHES FROM CLASSIC PROBABILITY THEORY

Because many earthquakes can happen every week (or even every day) in seismic-prone regions and more than one earthquake may occur on exactly the same day, our discussion will first focus on only one significant event per year taking place on only one day of the year. Other assumptions include the exclusion of leap years, the independence of events, and the stipulation that no aftershocks be taken into account. We note that the assumption of the occurrence of earthquakes independent of calendar days is intuitively reasonable, but it is also shown to be true in the literature (Hough, 2018).

We explore several well-known models for discrete random variables for 17 hypothetical probabilistic events E_i, defined below, in which $i = 1, 2, \ldots, 17$.

Let us first consider an urn model with 365 balls (one for each day of the year). All the balls are black, except for one red ball, which accounts for the chance of a significant earthquake happening at a specific selected day of the year. We define a "significant" earthquake S_E as an event that potentially could cause extensive damage and fatalities at a given site and that is determined quantitatively as per Jaimes and García-Soto (2018). Although significant earthquakes occur rarely, it is assumed for now, for the sake of establishing useful probabilistic models, that one S_E happens every year. So, for any given year the event E_1 = "S_E occurs on 19 September" can be modeled by extracting a ball from the urn, and the associated probability of a significant earthquake on the referred date (equal to the probability of extracting the red ball) is $P(E_1) = 1/365$.

If the ball is placed back in the urn and two Bernoulli trials are performed, the probability of E_2 = "Two S_E occur on 19 September of two consecutive years" is

$$P(E_2) = (1/365) \times (1/365) = 7.51 \times 10^{-6}.$$

Other experiments could include E_3 = "In the first and the last out of three consecutive years S_E occur on 19 September", the probability of which results in

$$P(E_3) = (1/365) \times (364/365) \times (1/365) = 7.49 \times 10^{-6}$$

and E_4 = "In a period of 33 yrs (e.g., from 1985 to 2017) S_E occur in the first and the last years on 19 September" with

$$P(E_4) = (1/365) \times (364/365)^{31} \times (1/365) = 6.89 \times 10^{-6}.$$

Changing the perspective, but considering again two S_E and a period of 33 yrs (except that the occurrence is not for predetermined years), the binomial distribution (Benjamin and Cornell, 1970) can be used to obtain the probability of E_5 = "In a period of 33 yrs (e.g., from 1985 to 2017) S_E occur in any of two years among the 33 on 19 September" with

$$p_Y(y) = \frac{n!}{y!(n-y)!} p^y (1-p)^{n-y} \quad y = 0, 1, 2, \ldots, n \qquad (1)$$

in which $n = 33$, $y = 2$, and $p = 1/365$; therefore

$$P(E_5) = \frac{33!}{2!(33-2)!}(1/365)^2(1-1/365)^{33-2} = 3.64 \times 10^{-3}.$$

Analogous examples to E_5 can be performed, for instance, using $n = 56, 61, 108$, and 563 (E_6, E_7, E_8, and E_9, respectively). These values correspond to the number of years associated to the historical events described in the Introduction. As a reminder, we briefly mention that they are the number of years since the first earthquake that struck what is known today as Mexico City is reported in historical records to be 563 yrs ago, since the first seismological observatory was inaugurated in Mexico City (108 yrs), since the 1957 seismic event occurred (61 yrs) and since the first accelerometer was installed at the CU station (56 yrs). Using the time windows selected above for these historical events, their probabilities are also reported in Table 1.

From another perspective, if the probability of the number of years until a second S_E happens on 19 September is of interest, the negative binomial distribution could be used, which is given by Benjamin and Cornell (1970)

$$p_{W_k}(w) = \binom{w-1}{k-1} p^k (1-p)^{w-k} \quad w = k, k+1, k+2, \ldots \qquad (2)$$

in which, for the referred example, $k = 2$ and $p = 1/365$. If for example the probability of E_{10} (or E_{11}) = "After exactly 56 (or 61) yrs two S_E occurred on 19 September" and E_{12} (or E_{13}) = "After exactly 108 (or 563) yrs two S_E occurred on 19 September," the corresponding probabilities are obtained using $w = 56, 61, 108$, and 563, respectively, in the previous equation and are listed in Table 1.

Moreover, the events E_{14} (or E_{15}) = "Two S_E occurred on 19 September in the first two years, or the first three years, or the first four years, …or the first 56 (or 61) yrs" and E_{16} (or E_{17})= "During either of the first two, or first three, or first four,… or first 108 (or 563) yrs, two S_E occurred on 19 September," can be computed using the cumulative mass function of the negative binomial distribution that is simply obtained by adding the probabilities for $w = 2, 3, 4, \ldots, 56$ (or 61) and for $w = 2, 3, 4, \ldots, 108$ (or 563), respectively. This leads to the values reported in Table 1.

As expected, the probabilities in Table 1 increase with the increasing number of considered years for the binomial and negative binomial distributions. This will be discussed in more detail in the Results and Discussion section.

Another probability distribution that will be especially useful later is the geometrical distribution (Benjamin and Cornell, 1970); assuming independence of the trials (as has

Table 1
Probabilities for the Events E_i by Considering the Return Period of Earthquakes

Event i	$P(E_i)$	$P(E_{iT})$	$\text{ratio}_T = P(E_i)/P(E_{iT})$	$P(E_{iT})$ Using p_{alt}	Probability Model
1	2.74×10^{-3}	2.57×10^{-4}	10.64	2.41×10^{-4}	B
2	7.51×10^{-6}	6.6303×10^{-8}	113.27	5.8164×10^{-8}	B
3	7.49×10^{-6}	6.6285×10^{-8}	112.99	5.8150×10^{-8}	B
4	6.89×10^{-6}	6.5776×10^{-8}	104.75	5.7731×10^{-8}	B
5	3.64×10^{-3}	3.4729×10^{-5}	104.81	3.0482×10^{-5}	Bi
6	9.97×10^{-3}	1.01×10^{-4}	98.71	8.8414×10^{-5}	Bi
7	1.168×10^{-2}	1.20×10^{-4}	97.33	1.05×10^{-4}	Bi
8	3.24×10^{-2}	3.73×10^{-4}	86.86	3.28×10^{-4}	Bi
9	2.548×10^{-1}	9.08×10^{-3}	28.06	8.03×10^{-3}	Bi
10	3.56×10^{-4}	3.5963×10^{-6}	98.99	3.1577×10^{-6}	NBi
11	3.83×10^{-4}	3.9182×10^{-6}	97.74	3.4406×10^{-6}	NBi
12	6×10^{-4}	6.9033×10^{-6}	86.91	6.0665×10^{-6}	NBi
13	9.05×10^{-4}	3.225×10^{-5}	28.06	2.8551×10^{-5}	NBi
14	1.048×10^{-2}	1.01×10^{-4}	103.76	8.88×10^{-5}	NBc
15	1.234×10^{-2}	1.20×10^{-4}	102.83	1.05×10^{-4}	NBc
16	3.581×10^{-2}	3.76×10^{-4}	95.24	3.30×10^{-4}	NBc
17	4.565×10^{-1}	9.553×10^{-3}	47.79	8.412×10^{-3}	NBc

B, Bernoulli; Bi, binomial; NBi, negative binomial; NBc, negative binomial (cumulative); $P(E_i)$, the probability associated with an occurrence rate of one significant earthquake per year; $P(E_{iT})$, the probability associated with an occurrence rate linked to the return period referred in the article, as a result of seismic hazard and risk analyses; ratio_T, the ratio of $P(E_i)$ and $P(E_{iT})$; p_{alt}, exact occurrence rate from seismic hazard analysis.

been assumed here) and a constant value of p, the distribution of the number of trials N to the first success is defined by

$$p_N(n) = (1-p)^{n-1}p \qquad n = 1, 2, \ldots \tag{3}$$

The most relevant aspect of this geometrical probability distribution (to be taken advantage of later) is its relation to the return period that is equal to the inverse of p (i.e., $T_R = 1/p$); the simple idea explained by Gumbel (1958) ("This result is self-evident: if an event has a probability p, we have to make on average $[1/p]$ trials in order that the event happens once") can formally be proven to be the expected value of equation (3) (Jordaan, 2005)

$$E[N] = \frac{1}{p} = T_R. \tag{4}$$

In the next part of this article, we use significant earthquakes from interplate, intermediate depth, and shallow-crustal events (Jaimes and García-Soto, 2018) that could occur with certain characteristics of wave motion. In the event that they struck Mexico City, the motion would cause damage and fatalities.

We mention here that no consideration was taken of possible new code requirements, upgrading of structures, mitigation measures, etc., and one may expect a reduction of damage and deaths in future earthquakes under such circumstances. This and the fact that information is scarce, if existent at all, led us to consider only the more recent catastrophic earthquakes in relating deaths with seismic intensity (Jaimes and García-Soto, 2018).

SEISMIC HAZARD, DEATHS STATISTICS, AND SIGNIFICANT EARTHQUAKES

A significant earthquake was defined above as an event associated with extensive damage and fatalities at a given site, and it was mentioned that this is determined quantitatively as per Jaimes and García-Soto (2018). Here, we give a brief summary of the article by Jaimes and García-Soto to contextualize the concept of significant earthquake that we use, but the interested reader is referred to the actual article for more information (Jaimes and García-Soto, 2018). A significant earthquake S_E is a seismic event generating a pseudospectral acceleration (SA) for $T = 2$ s equal to or larger than 30 cm/s/s at the CU station (firm soil) in Mexico City that could be generated by focal mechanisms corresponding to either interplate, intermediate depth, and shallow-crustal earthquakes from seismic zones used for estimating the seismic hazard at Mexico City. For readers not familiar with the previous terminology, a pseudospectral acceleration can be roughly understood as the acceleration that a hypothetical simple structure (an "inverted pendulum" with certain mass and rigidity) would exhibit at a given site in Mexico City, due to the action of a significant (large) earthquake that can be associated with a given return period, as explained below (i.e., how likely it is that such a large earthquake will occur in any given year). For the nonexperts in seismology, rather than considering the term "pseudospectral acceleration" or trying to understand the preceding paragraph, a significant earthquake can be simply understood as a large earthquake capable of causing extensive damage and fatalities at the site of interest.

The selected intensity can be considered as a lower bound for potentially damaging earthquakes affecting Mexico City that could cause fatalities (Jaimes and García-Soto, 2018). The reason for the selected intensity measure and the CU station is that it was found to correlate adequately in terms of the number of deaths, and it was decided that it could be used at a regional level (for the whole city). Apart from the seismic hazard from the mentioned types of earthquakes, data on loss of life were compiled for several recent and historic events causing severe damage and deaths in Mexico City (Jaimes and García-Soto, 2018), leading to proposing an expression to relate loss of human life with the selected seismic intensity and which is the basis of the annual exceedance rate and related return period described below.

We note that the discussion in the preceding part of this article was restricted to only one S_E occurring each year; however, the occurrence rate for Mexican interplate, intermediate depth, and shallow-crustal earthquakes affecting Mexico City is smaller, as can be shown by performing a probabilistic seismic hazard assessment (PSHA), including the three types of mentioned earthquakes (Jaimes and García-Soto, 2018), resulting in an annual exceedance rate $\nu(\mathrm{im})$ for 30 cm/s/s equal to 0.094 for the three types of earthquakes. As mentioned below, this rate can be associated with a return period, and it is all the reader not familiar with PSHA needs to know (plus some basic knowledge of probability theory) to reproduce the probabilities of occurrence of two earthquakes reported later. The above-mentioned rate means that the return period for this intensity is $T_R = 1/0.094 = 10.64$ yrs. For the present discussion, if this is related to the geometric probability distribution referred to earlier, we obtain that $p = 0.094$ is the probability of the exceedance given $\mathrm{SA}(T = 2\ \mathrm{s}) = 30$ cm/s/s for a time interval of one year, which will be assumed as the probability that an earthquake generating at least the selected intensity can occur in any given year. Furthermore, the chances that such an earthquake occurs any given day are now $(1/365) \times (1/10.64)$ (instead of 1/365 as in the previous analysis). Therefore, the probabilities of E_1, E_2, E_3, and E_4 can be recalculated using $(1/365) \times (1/10.64)$ instead of 1/365, and $1 - (1/365) \times (1/10.64)$ instead of 364/365. Moreover, the probabilities for E_5 to E_9 can be recalculated by substituting $p = (1/365) \times (1/10.64)$, and the same applies for E_{10} to E_{17}. We report the results of this restatement in the last part of this article.

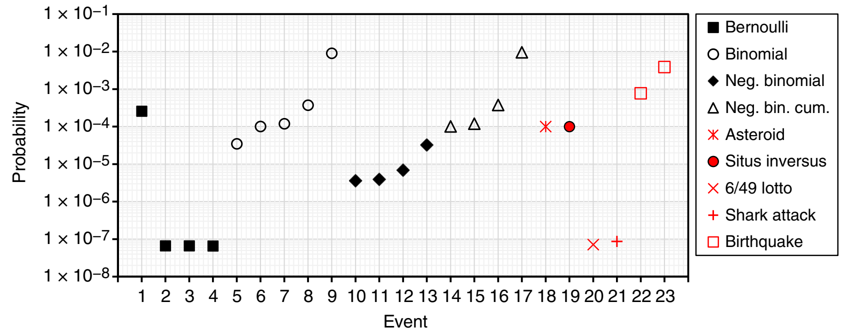
The selected annual exceedance rate $\nu(\mathrm{im})$ of a ground-motion intensity measure of value im (i.e., pseudospectral acceleration) accounts for the contribution to seismic hazard of all of the considered earthquakes (i.e., interplate, intermediate depth, and shallow crustal). If the probabilities associated with each type of earthquake are desired, they can be calculated by noting that, $\nu(\mathrm{im}) = \nu(\mathrm{im})_{\mathrm{Inter}} + \nu(\mathrm{im})_{\mathrm{InDepth}} + \nu(\mathrm{im})_{\mathrm{crust}} = 0.082 + 0.011 + 0.001 = 0.094$ corresponding to the annual exceedance rates for interplate, intermediate depth, and shallow-crustal earthquakes, respectively. The readers can recognize from the preceding sentences that there are different types of earthquakes, and they may guess that they affect structures in a different way (and so the need for a distinction), which is indeed the case. Although not pursued in the present article, it would be instructive to write an EduQuakes column in the future to convey to students and the public both the characteristics of the different types of earthquakes and information about which types of buildings and infrastructure are more vulnerable to the effects of each type of earthquake.

Because we are interested in the probability of the exceedance given $\mathrm{SA}(T = 2\ \mathrm{s}) = 30$ cm/s/s for a time interval $t = 1$ yr, we consider that the process of exceedance (like that assumed for the earthquakes' occurrence) is a Poisson process (i.e., stationarity, nonmultiplicity, and independence are considered; Benjamin and Cornell, 1970), and it can be shown that $P(\mathrm{SA} > 30\ \mathrm{cm/s/s}) = 0.088$, which is the exact value of the probability of exceedance; this can be obtained with a conditional probability of a ground-motion intensity measure over a certain threshold conditioned on a time interval; however, we do not describe it here in detail. We rather note that this value is very similar to $\nu(\mathrm{im}) = 0.094$, and either one of these values is used in the next part of the article to compute the desired probabilities (about two S_E) and to discuss the impact of using different exceedance rates (and thus different return periods, as will be seen). Nevertheless, to point out the relation between the exceedance rates and a Poisson process (and to introduce another well-known probabilistic model in this article), it is noted that the probability of exceedance equal to 0.088 is practically equivalent to the probability of observing exactly one large earthquake (i.e., one S_E generating $\mathrm{SA}(T = 2\ \mathrm{s}) > 30$ cm/s/s) in one year, if using $\lambda = \nu(\mathrm{im}) = 0.094$, $t = 1$ yr, and $x = 1$ in the Poisson distribution given below (Benjamin and Cornell, 1970):

$$P_X(x) = \frac{(\lambda t)^x}{x!} e^{-\lambda t} \quad (5)$$

RESULTS AND DISCUSSION

We use the probabilistic models described above to obtain the final results in this section but also by considering the probability discussed previously on the occurrence of one S_E in a given year (i.e., the probability that a significant seismic event generates $\mathrm{SA}(T = 2\ \mathrm{s}) > 30$ cm/s/s for any given year at the site of interest). In particular, we employ the return period for such events, $T_R = 10.64$ yrs (i.e., $p = \nu(\mathrm{im}) = 1/T_R$) to restate the chances that such an earthquake occurs on any given day of the year as $(1/365) \times (1/10.64)$. With this new value of $p = (1/365) \times (1/10.64)$, all the events E_i, the probabilities of which were computed above, are to be used to compute the probabilities for the same events but with the



▲ **Figure 1.** Probabilities computed in this study for the events E_i and others. The color version of this figure is available only in the electronic edition.

new p-value. They are named $P(E_{iT})$, and we list them in Table 1; the inclusion of "T" in the subscript is for emphasizing that the probabilities are restated for the considered return period.

The results in Table 1 show that the probability of the occurrence of two significant earthquakes on the same date in two years striking the same site is quite low for all the considered scenarios discussed above. This is not necessarily expected, if it is considered that the probability of the occurrence of a significant event, without the restriction of being on a specific date, is relatively high (approximately 9% any year) for Mexico City, which is an earthquake-prone region. If the chances of such event happening on any given day (e.g., on 19 September) are introduced, however, this probability decreases to about 0.026% (i.e., the $P(E_{1T})$ value in Table 1), and the probabilities of the events involving two earthquakes occurring on the same date are even smaller (on the order of 10^{-5} to 10^{-8}, depending on the selected event), unless large periods of observation are considered (over a century).

We point out that E_6–E_9 (binomial) and E_{14}–E_{17} (negative binomial) lead to comparable results (see Fig. 1, discussed below) and that, in general, $P(E_i) \times (1/10.64) \neq P(E_{iT})$, as can be observed in Table 1, in which the ratios, $\text{ratio}_T = P(E_i)/P(E_{iT})$, are listed and differ from T_R; note however, that the relationship $P(E_{iT}) \approx P(E_i)/T_R^2$ does lead to a reasonably good approximation in many cases. This is possibly linked to the fact that $n = 2$ (two years) is used in the binomial coefficients, leading to a term $p^2 = 1/T_R^2$; this term is involved in probabilities $P(E_2)$, $P(E_3)$, and $P(E_4)$, too. The approximation is less adequate for increasing numbers of considered years, and it does not hold for very long observation periods (e.g., more than five and a half centuries).

As mentioned earlier, the computed probabilities increase with an increasing number of considered years for the binomial and negative binomial distributions, as can be observed in Figure 1, in which all the probabilities computed in this study are depicted; the increasing trend is shown by the increasing probability values from $P(E_{5T})$ to $P(E_{9T})$ (events 5–9; circles) for the binomial distribution, and from $P(E_{10T})$ to $P(E_{13T})$ (events 10–13; solid diamonds) for the negative binomial distribution, or from $P(E_{14T})$ to $P(E_{17T})$ if the cumulative case of the negative binomial distribution is considered (events 14–17, triangles). The vertical axis in Figure 1 is in logarithmic scale for a better observation of the probabilities.

Let us now consider that for a set of large earthquakes, the focus is on a subset of earthquakes that already caused damage and deaths in Mexico City. From this subset, we are interested in the probability that two or more have the same "birthday" (i.e., they occurred on 19 September); this probability can be computed by modifying an equation for "birthdays" coincidence (Jordaan, 2005)

$$p_{\text{Birthquake}} = 1 - \left[\left(1 - \frac{1}{365 \times T_R}\right) \times \left(1 - \frac{2}{365 \times T_R}\right) \times \ldots \times \left(1 - \frac{r-1}{365 \times T_R}\right)\right], \quad (6)$$

in which r is the number of earthquakes in the subset referred above and T_R is the return period.

Probability values calculated from equation (6) are equal to 7.72×10^{-4} and 3.86×10^{-3} for r equal to 3 (events with more fatalities in Jaimes and García-Soto, 2018) and 6 (all events with fatalities in Jaimes and García-Soto, 2018), respectively. Such values are not far from some probabilities reported in Table 1 for $P(E_{iT})$, as observed in Figure 1 (events 22 and 23), which is interesting, given the simplicity of equation (6) and the not-so-dissimilar results if compared with some $P(E_{iT})$s. However, empirical information on rather rare seismic events is required for using equation (6), which also accounts for the cumulative chances of having three (or three to six) S_E on the same date (for the selected subsets). Nonetheless, it is a valuable alternative and a straightforward way to obtain a reference value, without the need of performing PSHA or other complex analyses. To draw a parallel between humans and earthquakes for this type of probabilistic estimation, it is as if one century in human life were equivalent to 10 yrs for the "earthquake births" (or as if one millennium in human life were equivalent to one century for the "earthquake births"), because a birthday is celebrated every year, and a significant earthquake occurs roughly every 10 yrs (for Mexico City and the seismic conditions described above).

We also note that $P(E_{iT})$ for events $i = 5$–9 can also be approximated using the Poisson distribution (introduced earlier), resulting in probabilities $P(E_{iT})$ equal to 3.58×10^{-5}, 1.02×10^{-4}, 1.21×10^{-4}, 3.76×10^{-4}, and 9.09×10^{-3} for $i = 5$–9, respectively. These values are similar to those reported in Table 1; which means that, for the values used here, the Poisson distribution approximates closely the binomial distribution (the reader may contrast this with the more common example in probability books, in which the binomial distribution is approximated by the normal distribution).

Similarly, aimed at computing $P(E_{iT})$ for events $i = 14$–17, the distribution of X_k, the time to the kth arrival of a Poisson process, is modeled by the gamma distribution, the probability density function of which is given by the following equation (Benjamin and Cornell, 1970):

$$f_{X_k}(x) = \frac{\lambda(\lambda x)^{k-1}}{\Gamma(k)} e^{-\lambda x} \quad x \geq 0, \tag{7}$$

in which $\Gamma(\cdot)$ is the gamma function, and considering $\lambda = (1/365) \times (1/10.64)$, $k = 2$, and $n = 56$, 61, 108, and 563, the cumulative distribution function can be obtained by integration as follows:

$$\int_0^n f_{X_k}(x)dx, \tag{8}$$

in which the upper limit corresponds to the selected times in years. By performing the integration, the obtained probabilities $P(E_{iT})$ result in 1.03×10^{-4}, 1.22×10^{-4}, 3.8×10^{-4}, and 9.545×10^{-3} for $i = 14$–17, respectively. These values are also very similar to those reported in Table 1 (and actually to the values computed using the Poisson distribution), which means that, analogously to the previous case, the gamma distribution approximates adequately the negative binomial distribution.

The reader may wonder why different probabilistic models (and indeed different probability values) are used in the study, as reported in Table 1. The answer is that the probabilistic model depends on the event E_i. For instance, for events E_2–E_4, the product of the Bernoulli trials is enough, because the exact years of the occurrence for the earthquakes in the given date are fixed, unlike events E_5–E_9, in which the events for the given date can happen in any pair of years within the considered time interval (although they are limited to exactly two earthquakes on the same day). Therefore, there is no point in trying to assess which distribution is better or more convenient, because they are related to different events (even though all of them involve two significant earthquakes occurring on the same date in different years and striking at the same site). Nevertheless, a conclusion that can be drawn is that for all the considered probabilistic models and time periods (except for many centuries), the events involving two significant earthquakes for the described conditions are highly improbable.

To explore other differences, consider now that the value for the probability of exceedance described at the end of the previous part of this article is used [i.e., $P(\mathrm{SA} > 30\ \mathrm{cm/s/s}) = 0.088$, instead of 0.094, is employed]. $p_{\mathrm{alt}} = (1/365) \times (1/11.36)$ is used this time (this is equivalent to considering a longer return period equal to 11.36 yrs). The use of p_{alt} does have a moderate impact on decreasing the probabilities, as can be observed in Table 1; however, the results are comparable to those previously computed. These results also imply that longer return periods (e.g., in zones with not so significant seismic activity) will lead to even smaller probabilities for the considered events. Naturally, another decision that will decrease the probabilities even further is the selection of a higher-intensity threshold, for example, by considering only seismic events linked to many deaths (that will be associated with longer return periods). Another paper could be used to explore the range of results for any desired limit (Jaimes and García-Soto, 2018), because it is beyond the scope of this article.

To close the discussion in this article, we compare the obtained probabilities with others reported in the literature for events of a very different kind. For instance, a probability of 1×10^{-4} (1-in-10,000 chance) within a century was estimated for an ~2-km-diameter comet or asteroid colliding with the Earth (Chapman and Morrison, 1994). The probabilities of the events in Table 1 associated with a time period of 108 yrs are larger but of the same order of magnitude, being around 3.7×10^{-4}, and if 61 yrs is considered, the chances of a comet colliding with the Earth and of two earthquakes striking the same site on the same date are around the same (see Fig. 1; events 7, 15, and 18). Other events in Table 1 are far more unlikely than an asteroid collision (or than a person having a heart on the right side, a situs inversus, with a probability of 1×10^{-4} [event 19 in Fig. 1]; Burn, 1991). Another example is the probability of winning the 6/49 lotto (Ariyabuddhiphongs, 2011; event 20 in Fig. 1), 7.143×10^{-8} (a one in 14 million chance) that is even larger than $P(E_{2T})$, $P(E_{3T})$, and $P(E_{4T})$ in Table 1; that is, E_2, E_3, and E_4 are more unlikely than winning the lottery (and also than being attacked by a shark, with a probability of 8.7×10^{-8}; International Shark Attack File, 2001; event 21 in Fig. 1). As cited for each case, the previous low probabilities for nonearthquake-related events are also included in Figure 1, so that the previous comparisons can be readily observed in the figure.

We end this discussion by noting that the previous comparison may not be an exercise in futility; it could actually be employed to communicate in a clear way to the public how improbable the event studied here is but that nonetheless it happened, even though it is an extremely rare occurrence! This should bring awareness for natural disaster management, prevention, and mitigation agencies to promote 19 September as a commemoration day for all victims of natural disasters, as well as a victims' Remembrance Day for the 1985 and 2017 earthquakes.

CONCLUSIONS

On the afternoon of 19 September 2017, just over a couple of hours after an earthquake-awareness drill, an intense earthquake struck Mexico City and neighboring states, causing death and destruction. This earthquake shocked the Mexican people, not only because of its devastating and traumatic effects, but also because it occurred exactly on the same date as the previous well-known 1985 catastrophic earthquake that was still in the memory of many Mexicans.

In this study, discussion is given about how this rare coincidence could have implications in seismic engineering, disaster management, and possibly in other areas related to social sciences. The discussion elaborates also on how the computing of probabilities of significant earthquakes occurring on given days may be important, if it is considered that certain days may be more critical in terms of seismic risk, especially public holidays when people are gathered in large numbers in structures that may be vulnerable to earthquakes.

The occurrence of significant earthquakes on specific dates (including the occurrence of another event on the same date) could also be important from the standpoint of other issues intuitively considered relevant, such as psychological effects or other social aspects. The computing of the probability of two seismic events causing damage and death at the same site on the same day of different years, using formal tools from classic probability theory and PSHA, has been demonstrated and described so as to be understandable for students and the public in general. Several probability models were extracted from classical probability theory to establish the probability of events: the binomial, negative binomial, geometric, exponential, Poisson, and Gamma distributions, plus other models, including a proposal by modifying one of them, which could be instructive to young students of engineering and other fields.

It is concluded that the probabilities of two significant earthquakes striking the same site in different years, on the same day, are very low, which might be unexpected, considering that the occurrence of significant seismic events is not unusual in Mexico City (around 9% chance in any given year). If the analysis includes the restriction that such significant event could happen on any given day (e.g., on 19 September), this probability decreases to ∼0.026%, leading to probabilities of the events involving two earthquakes being even smaller (on the order of 10^{-5} to 10^{-8}), unless periods of observation of more than a century are assumed. These probabilities can also be lower than, for instance, the probability of an ∼2-km-diameter comet or asteroid colliding with the Earth, a person having a heart on the right side, the probability of winning the 6/49 lotto, or of being attacked by a shark. It is considered that such a result could be used to communicate to the public how improbable the referred-to event is, and to bring awareness on natural disaster management, prevention, and mitigation by promoting 19 September as a commemoration day for all victims of natural disasters, as well as specifically the victims of the 1985 and 2017 earthquakes.

DATA AND RESOURCES

The ground-motion records from the CU station used for the seismic hazard assessment in Mexico City are the result of instrumentation and processing carried out by the Unit of Seismic Instrumentation of the Institute of Engineering UNAM. CENAPRED (2017). https://datos.gob.mx/busca/dataset/puntos-de-evaluacion-estructural-del-sismo-19-s (last accessed October 2018).

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Miguel A. Jaimes
Instituto de Ingeniería
Universidad Nacional Autónoma de México
Coordinación de Ingeniería Estructural. Circuito Interior
Ciudad Universitaria
Del. Coyoacán
04510 Mexico City, Mexico
mjaimest@iingen.unam.mx

Adrián D. García-Soto
Departamento de Ingeniería Civil
Universidad de Guanajuato
Campus Guanajuato
Sede Belén, Av. Juárez 77
Zona Centro, C.P.
36000 Guanajuato
Mexico
adgarcia@ugto.mx

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